

Cramers Rule

$$x_i = \frac{\det(A_i)}{\det(A)}$$

where: $A_i = [A]$ with the i th column replaced with the $[B]$ matrix/vector

example:

$$3x + 5y = 19$$

$$2x + 3y = 12$$

equation form

let: $x_1 = x$
 $x_2 = y$

so:
$$\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 19 \\ 12 \end{bmatrix}$$

matrix form

$x_1 = \frac{\det(A_1)}{\det(A)}$

$\swarrow$ 1st value

$\swarrow$ 1st column replaced with $[B]$

$$= \frac{\begin{vmatrix} 19 & 5 \\ 12 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix}} = \frac{(19)(3) - (5)(12)}{(3)(3) - (5)(2)} = 3$$

$x_2 = \frac{\det(A_2)}{\det(A)}$

$\swarrow$ 2nd value

$\swarrow$ 2nd column replaced with $[B]$

$$= \frac{\begin{vmatrix} 3 & 19 \\ 2 & 12 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix}} = \frac{(3)(12) - (19)(2)}{(3)(3) - (5)(2)} = 2$$